Hybrid Resource Allocation Problems - A Laboratory Case Study

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# Synopsis

The laboratory case study to be reported in this paper has been initiated by shipyard industry. Ship-building requires a large number of welding seems to be carried out by a relatively small number of co-operating automated robots. In the regular operation mode each robot is working with the same velocity. However, real world disturbances will cause varying velocities. Therefore rescheduling should be done such that one robot is used for assisting another one. A laboratory model of such a welding plant has been built up with two co-operating multi-link robots.

## Notation

$\mathcal{L},~\mathcal{R}$	sets of welding seems allocated to robots $L$ (left) and $R$ (right), respectively
$y_L^+, y_R^+  w_L(t), w_R(t)$	total lengths of welding seems allocated to robots $L$ and $R$ , respectively reference lengths of welding seems to be completed by robots $L$ and $R$ at time $t$ , respectively
$y_L(t), \ y_R(t)$	actual lengths of welding seems completed by robots $L$ and $R$ at time $t$ , respectively
e(t)	error
$-e_{\min}$	lower threshold of $e(t)$
$e_{\max}$	upper threshold of $e(t)$
$x_L(t), x_R(t)$	geometrical positions of robots $L$ and $R$ , respectively
$R_{+}$	geometrical restriction
$\delta_j$	additive velocity disturbance
$d_{ m max}$	bound on disturbance

### 1 INTRODUCTION

In modern shipyard industry (1) large container ships are built in hierarchical steps. In the phase of construction the ship is subdivided in a number of blocks (Figure 1). Each of these blocks consists of a number of panels which on their part are composed of sheet metal which is cut from steel plates.

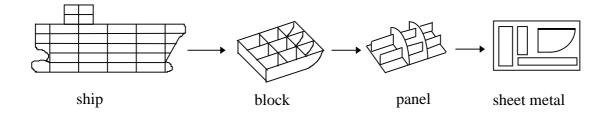


Fig 1 Segments of a ship

The laboratory case study to be reported in this paper addresses certain problems which arise in the automated welding process of a panel. Manufacturing is carried out in production cells, the control of which is integrated in the over-all production process (Figure 2). The geometrical data of the ship construction are obtained by means of CAD and are fed into a module for off-line programming of the robots. On the same level the production scheduler and the material flow system act on the cell control system.

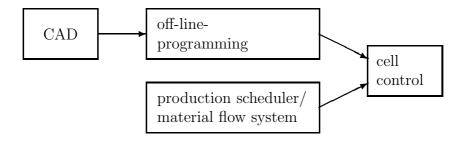


Fig 2 Integration of cell control

In the production cell the cut metal sheets must be brought into position, they must be fixed and then be welded. Only this welding process will be considered in the present paper. Therefore the tasks to be carried out are the welding seems, and the resources are robots to be allocated to these tasks. A reference schedule is set up by the off-line programming module based on apriory estimates of the welding velocity.

An unavoidable type of disturbances which is inherent to the welding process are inaccuracies in the size of the welding gap. They are due to inaccuracies of both the cutting, and the fixing process. In case of a large welding gap more welding material must be applied which requires a reduced welding velocity. Therefore only a mean value can be prescribed for the time needed to produce a welding seem of a given length. However, since the actual tolerances usually are known, an upper and a lower bound on the welding velocity is available.

Now subtask durations deviating from expected durations may cause significant delays in the over-all processing time. Therefore rescheduling should be done rather than keeping the regular operation mode. Rescheduling means that one robot is used for assisting annother one in order to compensate for an arissen delay in the progress of the task. Rescheduling aims at the following control goals:

- (1) Collision avoidance.
- (2) Minimization of the over-all processing time. Therefore it should be guaranteed that none of the robots becomes idle. This control goal implies task completion of each robot at nearly the same time.

## 2 LABORATORY EQUIPMENT

A laboratory model of such a welding plant has been built up (Figure 3) showing two cooperating multi-link robots. As can be seen on the photo both robots can move on a common horizontal rail, hence they can never pass each other. This is a geometrical restriction. Vertical bars have been attached to a vertical wall. Each bar is subdivided into serveral segments thus modelling the seems to be 'welded'.

Figure 4 outlines the structure of robot control. It contains all essential elements of industrial cell control: rescheduling, robot control, and documentation. For the sake of simplicity the route generation has been included in the control program. Hence rescheduling and route generation are implemented on a personal computer, whereas robot control itself is implemented by separate hardware.

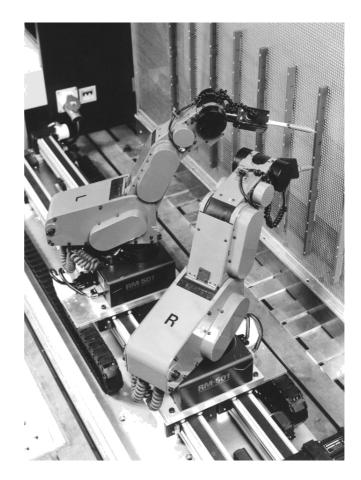


Fig 3 Two robot laboratory model

#### 3 A GENERAL RULE BASIS

Following the method presented in (2), (3), a general rule basis for discrete-event control of the above plant will be established. Let  $\mathcal{T}$  be the set of all welding seems to be produced,

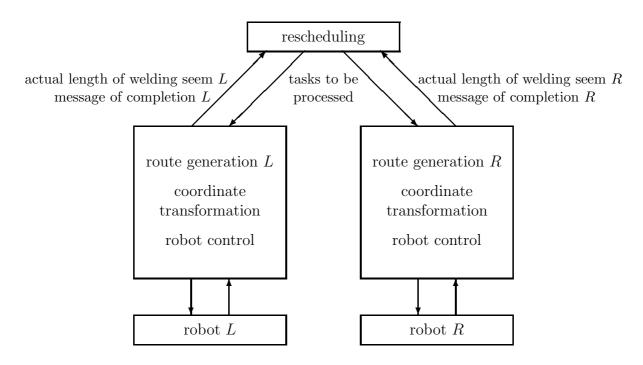


Fig 4 Control structure

and let  $\mathcal{T}$  be subdivided in disjoint subsets  $\mathcal{L}$  and  $\mathcal{R}$ ,  $\mathcal{T} = \mathcal{L} \cup \mathcal{R}$ , such that robots L (left) and R (right) are allocated to subsets  $\mathcal{L}$  and  $\mathcal{R}$ , respectively, in the nominal mode of operation. Let  $y_L^+$  and  $y_R^+$  be the total lengths of welding seems in the subsets  $\mathcal{L}$  and  $\mathcal{R}$ , respectively. Moreover, let  $w_L(t) \in [0, y_L^+]$  and  $w_R(t) \in [0, y_R^+]$  denote the reference lengths of welding seems to be completed within the subsets  $\mathcal{L}$  and  $\mathcal{R}$  at time t, respectively, given by the reference schedule. Since there will be no idle time in the reference schedule,  $w_L(t)$  and  $w_R(t)$  are strictly monotone, and hence the reference operation can be characterized by a unique and invertible mapping  $w \colon [0, y_L^+] \to [0, y_R^+]$ . Now let the process be subject to disturbances and let  $y_L(t) \in [0, y_L^+]$  and  $y_R(t) \in [0, y_R^+]$  be the actual lengths of welding seems completed by robots L and R at time t, respectively. Then the error  $e(t) = y_R(t) - w(y_L(t))$  will arise. In (2) thresholds  $-e_{\min}$  and  $e_{\max}$  for the error e(t) are defined where  $-e_{\min} < 0 < e_{\max}$ , and a rule basis for resource to task allocation is proposed which provides switches between three possible modes of operation:

- (i) Regular mode: Robots L and R are working on subsets  $\mathcal{L}$  and  $\mathcal{R}$ , respectively. Whenever a welding seem is completed, the error e(t) is compared with the above thresholds. In case of  $e(t) \geq e_{\text{max}}$  the mode will be switched from 'regular' to 'R-assists-L'. In case of  $e(t) \leq -e_{\text{min}}$  the mode will be switched from 'regular' to 'L-assists-R'.
- (ii) R-assists-L mode: Both robots are working on subset  $\mathcal{L}$ . Whenever  $e(t) < e_{\text{max}}$  upon completion of a welding seem by robot R, the mode will be switched to 'regular'.
- (iii) L-assists-R mode: Both robots are working on subset  $\mathcal{R}$ . Whenever  $e(t) > -e_{\min}$  upon completion of a welding seem by robot L, the mode will be switched to 'regular'.

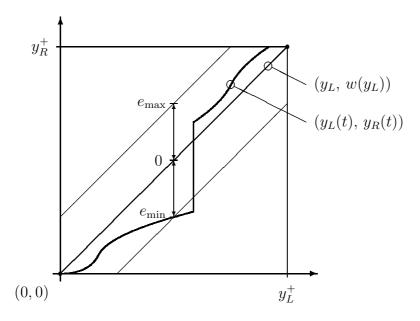


Fig 5 Reference trajectory and example of an actual trajectory

Figure 5 illustrates a typical situation by assuming  $y_L^+ = y_R^+$ . The straight line  $(y_L, w(y_L))$  is the nominal trajectory in the absence of any disturbance. The curve  $(y_L(t), y_R(t))$  is a typical trajectory of the disturbed plant subject to the above rule basis. The error e(t) falling short of threshold  $-e_{\min}$  does not cause immediate switch to the L-assists-R mode because robot L must complete its welding seem. The vertical part of the trajectory indicates the L-assists-R mode.

## 4 RULE BASIS FOR A CONCRETE GEOMETRY

The general rule basis given in section 3 does not enable a proper choice of thresholds  $-e_{\min}$  and  $e_{\max}$  nor does it enable specification of an admissible upper bound on the disturbance such that no robot gets idle. To this end geometrical restrictions and also the continuous dynamics of the welding process must be taken into account. Consider as an example the concrete geometry given in Figure 6. Let the subtasks numbered by 1 to 2n represent the welding seems to be processed. Let robots L and R be movable on a sliding carriage whose length is  $R_+$  and which itself is movable from left to right. In the nominal mode L is allocated to odd subtasks  $\mathcal{L} = \{1, 3, 5, \ldots, 2n-1\}$ , whereas R is allocated to even subtasks  $\mathcal{R} = \{2, 4, 6, \ldots, 2n\}$ . Both robots are processing from left to right. All subtasks are assumed to have the same length, and both robots are assumed to have the same nominal velocity which is normed to 1. Geometrical restrictions: L can never be on the right hand side of R, and the distance between L and R can never exceed  $R_+$ .

Figure 7 shows a section drawn from Figure 6 thus enabling a local view for establishing the concrete rule basis. It is assumed that in the regular mode all subtasks on the left of positions  $x_L$  and  $x_R$  have been completed, and all remaining subtasks are not yet processed. Therefore positions  $x_L$  and  $x_R$  correspond to the completed lengths  $y_L = x_L + (i-1)/2$  and  $y_R = x_R + (i-1)/2$ , respectively. Whenever L has completed subtask

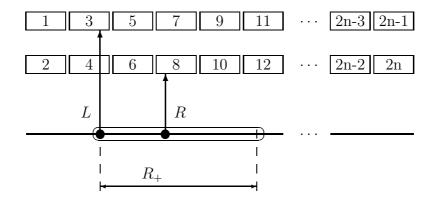


Fig 6 Considered geometry of subtasks

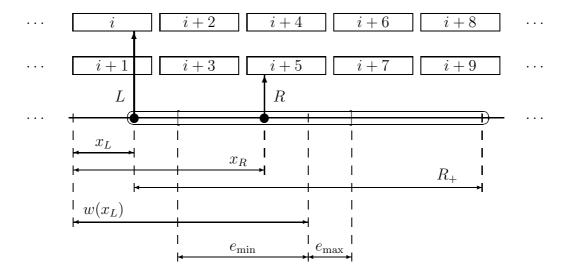


Fig 7 Local view on the subtasks

 $i, x_L$  and  $x_R$  are reduced by 1, and i is replaced by (i+2). This reflects the local view proposed above. For the local view the reference trajectory  $w(x_L) = x_L + R_+/2$  is near at hand, and hence the error becomes  $e(t) = x_R(t) - w(x_L(t))$ . The specification of thresholds  $-e_{\min}$  and  $e_{\max}$  will be discussed later in section 5. By assuming  $R_+ = 4$  (see Figure 7) the general rule basis proposed in section 3 now takes the following concrete form:

- (i') Regular mode: Robots L and R are working on subsets  $\mathcal{L}$  and  $\mathcal{R}$ , respectively. Whenever R has completed a subtask and  $e(t) \geq e_{\max}$  ( $e(t) \leq -e_{\min}$ ) then the mode will be switched from 'regular' to 'R-assists-L' ('L-assists-R').
- (ii') R-assists-L mode: Robot R completes its subtask and then proceeds with subtask i+4. Having completed this one, R proceeds with the next unprocessed subtask in R. Robot L processes subtasks i and i+2. Robot L may become idle until R has completed i+4. Having completed subtasks i, i+2 and i+4, the mode will be switched to 'regular'.

(iii') L-assists-R mode: Robot R skips its next subtask and proceeds with subtasks in  $\mathcal{R}$  following that one. Robot L having completed subtask i proceeds with the skipped subtask in  $\mathcal{R}$ . Having completed this one the mode will be switched to 'regular'.

## 5 ANALYSIS OF THE HYBRID CLOSED-LOOP SYSTEM

The continuous-time welding process can be modelled by an integrator relating the velocity to the length of the welding seem. The effect of the varying welding gap described in section 1 can be modelled by a position dependent additive disturbance  $\delta_j(\cdot)$  which is active while processing welding seem j, e. g. by robot L, whose motion can hence be described by

$$\dot{x}_L(t) = 1 + \delta_i(x_L(t)) .$$

Of course,  $\delta_j(\cdot)$  is not known beforehand. However, an upper bound  $d_{\text{max}}$ ,  $||\delta_j(\cdot)||_{\infty} \leq d_{\text{max}} < 1$ , can realistically be specified. So the above differential equation can be replaced by the differential inclusion

$$\dot{x}_L(t) \in [1 - d_{\text{max}}, 1 + d_{\text{max}}]$$
.

Therefore an upper and a lower bound on the processing time of each subtask can easily be determined. The same is of course valid for robot R. The time for switching from subtask to subtask and from one mode to annother one is neglected in the analysis of the over-all system.

Due to the communication between the discrete-event dynamics of the scheduler (controller) and the continuous-time dynamics of the welding process the over-all closed-loop system exhibits a hybrid nature. The interface is provided by the continuous comparison of continuous-time variables with fixed thresholds.

Among the many approaches which have been proposed for the analysis of hybrid systems during the past decade, *hybrid automata* (4) appear to be very suitable for the problem addressed here. The closed-loop system considered here can even be modelled by a *linear* hybrid automaton. This enables a computer aided analysis by means of available software-tools. So the following important questions can be analysed:

- (a) Given plant parameter  $d_{\text{max}}$  and design parameters  $-e_{\text{min}}$  and  $e_{\text{max}}$ , can compliance of the geometrical restrictions be guaranteed without a robot becoming idle?
- (b) Given plant parameter  $d_{\text{max}}$ , determine admissible values of  $-e_{\text{min}}$  and  $e_{\text{max}}$  provided they exist.

The answer to these questions can be given by computing the reachability set of the closed-loop system by means of the program 'HyTech' (5). For given  $d_{\text{max}} = 0.1$  the admissible thresholds  $-e_{\text{min}} = -1.4$  and  $e_{\text{max}} = 0.6$  have been obtained. For given  $d_{\text{max}} = 0.2$  the thresholds are  $-e_{\text{min}} = -1.1$  and  $e_{\text{max}} = 0.6$ . For larger disturbances the proposed scheduler cannot guarantee processing without idle phases.

### 6 VALIDATION BY LABORATORY EXPERIMENTS

The rule basis proposed in section 4 has been tested at the laboratory model with  $d_{\text{max}} = 0.2$ ,  $-e_{\text{min}} = -1.1$  and  $e_{\text{max}} = 0.6$ . One of many tested disturbance scenarios is reported in the following. Assume that all welding seems of  $\mathcal{R}$  are processed with velocity  $1 - d_{\text{max}}$ , whereas all welding seems of  $\mathcal{L}$  are processed with velocity  $1 + d_{\text{max}}$ . Figure 8 outlines the measured error e versus time t. During the time interval shown, the total of 2n = 40 welding seems have been processed. Three switches from the regular mode to the L-assists-R mode have been observed during the processing time. Figure 9 shows the distance between robots R and L versus time t. Obviously, the geometrical restrictions are met and so the design objective has been achieved.

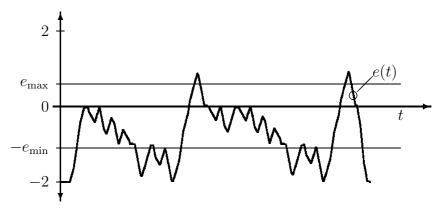


Fig 8 Measured error e versus time t

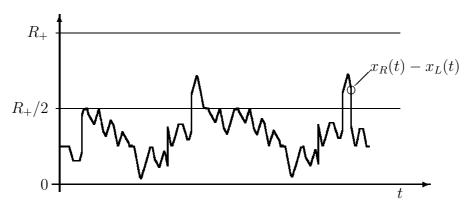


Fig 9 Distance between the robots versus time t

#### 7 CONCLUSION

A realistic scenario in modern shipyard industry has been investigated. Co-operating robots are processing a number of welding seems. The resulting resource to task allocation problem has been interpreted as a problem of designing a closed-loop control system, and its solution has been presented. It implies the design of a suitable discrete-event scheduler based on a general concept and leading to a rule basis which depends on two design

parameters. For a concrete geometry the mixed continuous-discrete model of the closed-loop system has been verified. This has been achieved by computing the reachability set using the available software 'HyTech'. The test of the scheduler at a laboratory equipment has shown, that the assumptions made for setting up the mathematical model were admissible.

### ACKOWLEDGEMENT

This work has been supported by the German Research Foundation (DFG).

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