A Hierarchical Control Architecture for Sequential Behaviours

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Abstract: This paper develops a hierarchical control architecture for so called *sequential behaviours*, i.e. for plant dynamics and specifications that are represented as formal languages of infinite-length words. Our main result is the elaboration of structural properties that (a) allow for abstraction based controller design and that (b) are preserved under closed-loop composition. Thus, we propose to alternate controller design, closed-loop composition and abstraction in order to construct a hierarchical control system. Technically, our results are based on a variation of input-output systems as introduced by Willems (1991), with a particular focus on *liveness properties*, i.e., sequential behaviours that are not necessarily *topologically closed*.

Keywords: discrete-event systems, ω-languages, sequential behaviours, hierarchical control.

1. INTRODUCTION

It is common engineering practice to address complex control problems by a hierarchical system design. In the context of supervisory control (Ramadge and Wonham, 1989), this principle has been formalized from a variety of perspectives, see e.g. Zhong and Wonham (1990); Wong and Wonham (1996); da Cunha et al. (2002); Leduc et al. (2005); Schmidt et al. (2008). In contrast to monolithic approaches, the benefits include not only a systematic derivation of adequate models for the individual levels of abstraction, but also computational feasibility for large scale systems.

In this paper, we further develop a hierarchical control architecture that was originally presented to address a class of hybrid systems (Moor et al., 2003) and subsequently discussed for discrete event systems (Perk et al., 2006). For each individual level of the hierarchy, it is proposed to design a controller according to a language inclusion specification. Regarding safety properties, one may base the design on an abstraction of the levels below, i.e., on a superset language that accounts for any trajectory the lower levels can evolve on. Clearly, an already available abstraction is the specification used for the design of the level below. Computational benefits are expected from alternating abstraction and controller synthesis, since the specification does not need to express how the control objective is achieved. This has been demonstrated by a transport system example in (Perk et al., 2008).

Regarding liveness properties, the situation is more involved than for safety properties. Here, (Moor et al., 2003; Perk et al., 2006) refer to a variation of input-output systems proposed by Willems (1991) in order to obtain a nonblocking closed-loop configuration as a structural liveness property. i.e., expressed independently of the particular plant dynamics. This setting appears to be a natural choice for the hybrid systems discussed in (Moor et al., 2003). However, more general liveness properties can be expressed by languages of infinite-length words, also known as *sequential behaviours* or *ω-languages*. Examples of liveness properties include eventual task completion or eventual feedback to a server of any individual one of a number of clients. The corresponding ω-languages share the technical property that they are not *topologically closed*. While the literature on supervisory control commonly represents relevant dynamics as $*$-languages, the core results are also available for ω-languages (Ramadge, 1989; Kumar et al., 1992; Thistle and Wonham, 1994), including the situation of partial observation (Thistle and Lamouchi, 2009) and abstraction based controller design (Moor et al., 2011). However, hierarchical control has not yet been addressed explicitly for general ω-languages.

In this paper, we extend the approach from (Moor et al., 2003) to the situation of not necessarily topologically closed ω-languages. While (Moor et al., 2003) is set within the framework of Willems' behavioural systems theory, and, thus, formally uses ω-languages to represent behaviours, it effectively requires plant and controller to be topologically closed, and, thus, excludes liveness properties other than a nonblocking closed-loop. Similarly, (Perk et al., 2006) models system components by prefix-closed $*$-languages, which exhibit a topologically closed limit, and, thus, can not represent general liveness properties. Conceptually, the current contribution still refers to a notion of *non-anticipating input-output systems* to achieve a nonblocking closed-loop configuration. However, we impose no further constraints regarding liveness properties possessed by the plant or required by the specification.

This paper is organized as follows. Section 2 introduces notation, recalls well-known facts and establishes required lemmas regarding formal languages. Section 3 discusses a closed-loop configuration with external signals and characterizes admissible controllers in terms of achievable closed-loop behaviour. Section 4 shows that relevant plant properties are retained under closed-loop composition, and, thereby, establishes a hierarchical control architecture.

2. PRELIMINARIES

For a *finite alphabet* Σ, the *Kleene-closure* Σ^* is the set of finite strings $s = \sigma_1\sigma_2\cdots\sigma_n$, $n \in \mathbb{N}$, $\sigma_i \in \Sigma$, including the *empty string*

$\varepsilon \in \Sigma^*$, $\varepsilon \notin \Sigma$. If for two strings $s, r \in \Sigma^*$ there exists $t \in \Sigma^*$ such that $s = rt$, we say r is a *prefix* of s, and write $r \leq s$. If $r \neq s$, we say r is a *strict prefix* of s and write $r < s$. A $*$*-language* over Σ is a subset $L \subseteq \Sigma^*$. The *prefix-closure* (or short *closure*) of $L \subseteq \Sigma^*$ is defined by $\mathrm{pre}\,L := \{r \mid \exists s \in L : r \leq s\} \subseteq \Sigma^*$. A language $L \subseteq \Sigma^*$ is called *closed*, if $L = \mathrm{pre}\,L$. Given $L,\ K \subseteq \Sigma^*$, we say K is *relatively closed w.r.t.* L if $K = \mathrm{pre}\,K \cap L$. The closure operator distributes over arbitrary unions of languages. However, for the intersection of two languages $L,\ K \subseteq \Sigma^*$, we have $\mathrm{pre}\,(L \cap K) \subseteq (\mathrm{pre}\,L) \cap (\mathrm{pre}\,K)$, and, if equality holds, L and K are said to be *non-conflicting*. A $*$-language K is said to be *complete*, if for all $s \in \mathrm{pre}\,K$ there exists $\sigma \in \Sigma$ such that $s\sigma \in \mathrm{pre}\,K$.

The *natural projection* $\mathrm{p_o} : \Sigma^* \to \Sigma_o^*$, $\Sigma_o \subseteq \Sigma$, is defined iteratively for $s \in \Sigma^*$, $\sigma \in \Sigma$: (1) $\mathrm{p_o}\varepsilon = \varepsilon$, (2a) $\mathrm{p_o}(s\sigma) = \mathrm{p_o}(s)\sigma$ if $\sigma \in \Sigma_o$, and (2b) $\mathrm{p_o}(s\sigma) = \mathrm{p_o}(s)$ if $\sigma \notin \Sigma_o$. The set-valued inverse $\mathrm{p_o}^{-1}$ is defined by $\mathrm{p_o}^{-1}(r) := \{s \in \Sigma^* \mid \mathrm{p_o}(s) = r\}$ for $r \in \Sigma_o^*$. When extended to languages, the projection distributes over unions, and the inverse projection distributes over unions and intersections. Furthermore, the closure operator commutes with projection and inverse projection. Given $L,\ K \subseteq \Sigma^*$, and a set of uncontrollable events $\Sigma_{uc} \subseteq \Sigma$, we say K is *controllable w.r.t.* (Σ_{uc}, L), if $((\mathrm{pre}\,K)\Sigma_{uc}) \cap (\mathrm{pre}\,L) \subseteq \mathrm{pre}\,K$. Given $L,\ K \subseteq \Sigma^*$, and a set of observable events $\Sigma_o \subseteq \Sigma$, we say K is *normal w.r.t.* (Σ_o, L), if $\mathrm{pre}\,K = (\mathrm{p_o}^{-1}\mathrm{p_o}\mathrm{pre}\,K) \cap (\mathrm{pre}\,L)$. Controllability, normality, completeness, prefix-closedness and relative closedness are retained under arbitrary union.

An *infinite string* over Σ is defined as a function $w : \mathbb{N} \to \Sigma$. By $\Sigma^\omega := \{w \mid w : \mathbb{N} \to \Sigma\}$ we denote the set of all infinite strings over Σ. A *monotone sequence of strings*, denoted by $(s_n) \subseteq \Sigma^*$, is a sequence $(s_n)_{n\in\mathbb{N}}$, $s_n \in \Sigma^*$, $s_n \leq s_{n+1}$ for all $n \in \mathbb{N}$. We call (s_n) unbounded if $|s_n|$ is unbounded. The point-wise *limit* of a monotone sequence (s_n) is denoted by $\lim\,(s_n) \in \Sigma^* \cup \Sigma^\omega$. An ω*-language* is a subset $\mathcal{L} \subseteq \Sigma^\omega$ and we denote ω-languages by calligraphic letters, in contrast to $*$-languages. The *prefix* of an ω-language is defined by $\mathrm{pre}\,\mathcal{L} := \{s \in \Sigma^* \mid \exists w \in \mathcal{L} : s < w\}$. The prefix of $w \in \mathcal{L}$ with length $n \in \mathbb{N}$ is denoted $w^n \in \Sigma^*$.

The prefix of any ω-language is complete and the prefix operator distributes over arbitrary unions of ω-languages. However, for the intersection of two ω-languages $\mathcal{L}, \mathcal{K} \subseteq \Sigma^\omega$, we have $\mathrm{pre}\,(\mathcal{L} \cap \mathcal{K}) \subseteq (\mathrm{pre}\,\mathcal{L}) \cap (\mathrm{pre}\,\mathcal{K})$, and, if equality holds, the languages are said to be *non-conflicting*. The languages $\mathcal{L},\ \mathcal{K} \subseteq \Sigma^\omega$ are *locally non-conflicting* if $(\mathrm{pre}\,\mathcal{L}) \cap (\mathrm{pre}\,\mathcal{K})$ is complete. For a language $L \subseteq \Sigma^*$, the *limit* is defined by $\lim L := \{\lim\,(s_n) \mid (s_n) \subseteq L\} \cap \Sigma^\omega$. Note that $\mathrm{pre}\lim L = L$, iff L is complete and prefix-closed. Hence, $\mathrm{pre}\lim\mathrm{pre}\,\mathcal{L} = \mathrm{pre}\,\mathcal{L}$.

Lemma 1. Given the languages $L,\ K \subseteq \Sigma^*$, with $K = \mathrm{pre}\,K$, then $\lim\,(L \cap K) = (\lim L) \cap (\lim K)$.

Proof. $(\subseteq)$: We pick $w \in \lim\,(L \cap K)$. Then, there exists monotone and unbounded (m.u.) sequence $(s_n) \in L$ and $(s_n) \in K$. Further, $w = \lim\,(s_n) \in \lim L$ and $w \in \lim K$. Thus, $w \in \lim L \cap \lim K$. $(\supseteq)$: We pick $w \in \lim L \cap \lim K$ and observe that for any $w^n \in L$ we have $w^n \in K$, for all $n \in \mathbb{N}$, since $K = \mathrm{pre}\,K$. Thus, $w^n \in L \cap K$, and $w = \lim\,(w^n) \in \lim\,(L \cap K)$. □

The *topological closure* (or short *closure*) of an ω-language $\mathcal{L} \subseteq \Sigma^\omega$ is defined by $\mathrm{clo}\,\mathcal{L} := \lim\mathrm{pre}\,\mathcal{L}$. An ω-language is said to be *closed* if $\mathrm{clo}\,\mathcal{L} = \mathcal{L}$. The limit of a prefix-closed $*$-language is topologically closed. Given two ω-languages $\mathcal{L},\ \mathcal{K} \subseteq \Sigma^\omega$, we say, that $\mathcal{K}$ is *relatively closed w.r.t.* $\mathcal{L}$, if $\mathcal{K} = (\mathrm{clo}\,\mathcal{K}) \cap \mathcal{L}$. The closure operator distributes over finite unions of ω-languages, see e.g. (Ramadge, 1989).

Let $(s_n) \subseteq \Sigma^*$ be a strictly monotone sequence of prefixes of a string $w \in \Sigma^\omega$. We define the *natural projection for infinite strings* by $\mathrm{p_o^\omega} w := \lim\,(\mathrm{p_o} s_n)$ for $w \in \Sigma^\omega, n \in \mathbb{N}, \Sigma_o \subseteq \Sigma$. The set-valued inverse is defined by $\mathrm{p_o}^{-\omega}(v) := \{\ w \in \Sigma^\omega \mid \mathrm{p_o^\omega}(w) = v\ \}$ for $s \in \Sigma_o^* \cup \Sigma_o^\omega$. When extended to ω-languages, the projection distributes over unions, the inverse projection over unions and intersections. Both commute with the prefix operator.

Lemma 2. Given the alphabets Σ, $\Sigma_o \subseteq \Sigma$ and the languages $L = \mathrm{pre}\,L \subseteq \Sigma^*$, $L_o \subseteq \Sigma_o^*$ and $\mathcal{L}_o \subseteq \Sigma_o^\omega$, then it is

(i) $(\mathrm{p_o^\omega}\lim L) \cap \Sigma_o^\omega = \lim \mathrm{p_o} L$,

(ii) $\mathrm{p_o}^{-\omega}\lim L_o = (\lim \mathrm{p_o}^{-1} L_o) \cap (\mathrm{p_o}^{-\omega}\Sigma_o^\omega)$,

(iii) $\mathrm{clo}\,\mathrm{p_o}^{-\omega}\mathcal{L}_o = (\mathrm{p_o}^{-\omega}\mathrm{clo}\,\mathcal{L}_o) \cup (\mathrm{p_o}^{-\omega}\mathrm{pre}\,\mathcal{L}_o)$.

Proof. Ad (i) $(\subseteq)$: We pick $w \in (\mathrm{p_o^\omega}\lim L) \cap \Sigma_o^\omega$. There exists $v \in \lim L$ s.t. $(\mathrm{p_o^\omega} v) \cap \Sigma_o^\omega = w$. Hence, there exists a m.u. sequence $(\mathrm{p_o} s_n) \in \mathrm{p_o} L$ such that $w = \lim\,(\mathrm{p_o} s_n) = \mathrm{p_o^\omega} v$. Thus, $w = \lim(\mathrm{p_o} s_n) \in \lim \mathrm{p_o} L$. $(\supseteq)$: We pick $w \in \lim \mathrm{p_o} L \subseteq \Sigma_o^\omega$. There exists a m.u. sequence $(s_n) \subseteq \mathrm{p_o} L$, s.t. $\lim\,(s_n) \cap \Sigma^\omega = \lim\,(s_n) \cap \Sigma_o^\omega = w$. Since L is prefix-closed, we can pick a m.u. sequence $(r_n) \subseteq L$ such that $(\mathrm{p_o} r_n) = (s_n)$. We denote its limit by $v := \lim\,(r_n) \subseteq \lim L \subseteq \Sigma^\omega$. Observe that $\mathrm{p_o}^{-\omega} v = \lim\,(\mathrm{p_o} r_n) \subseteq \mathrm{p_o^\omega}\lim L$. Thus, $w = \lim\,(\mathrm{p_o} r_n) \cap \Sigma_o^\omega \subseteq (\mathrm{p_o}^{-\omega}\lim L) \cap \Sigma_o^\omega$.
Ad (ii) $(\subseteq)$: We pick $w \in \mathrm{p_o}^{-\omega}\lim L_o \subseteq \mathrm{p_o}^{-\omega}\Sigma_o^\omega$. Then, $\mathrm{p_o^\omega} w \in \lim L_o$, i.e. we can pick a m.u. sequence $(\mathrm{p_o} s_n) \subseteq L_o$ and observe that $(s_n) \subseteq \mathrm{p_o}^{-1} L_o$. Hence, $w = \lim\,(s_n) \in (\lim \mathrm{p_o}^{-1} L_o) \cap (\mathrm{p_o}^{-\omega}\Sigma_o^\omega)$. $(\supseteq)$: We pick $w \in (\lim \mathrm{p_o}^{-1} L_o) \cap (\mathrm{p_o}^{-\omega}\Sigma_o^\omega)$. There exists a m.u. sequence $(s_n) \subseteq \mathrm{p_o}^{-1} L_o$, with $\lim\,(s_n) \cap (\mathrm{p_o}^{-\omega}\Sigma_o^\omega) = w$. The sequence $(\mathrm{p_o} s_n) \subseteq L_o$ is also m.u., since in the case of a bounded sequence, we must have $w = su$ with $u \in (\Sigma - \Sigma_o)^\omega$. In particular $w \notin \mathrm{p_o}^{-\omega}\Sigma_o^\omega$. We denote its limit by $v := \lim\,(\mathrm{p_o} s_n) \subseteq \lim L_o \subseteq \Sigma_o^\omega$ and observe that $\mathrm{p_o^\omega} w = v$. Hence, $w \subseteq \mathrm{p_o}^{-\omega}\lim L_o$.
Ad (iii) $(\subseteq)$: We pick $w \in \mathrm{clo}\,\mathrm{p_o}^{-\omega}\mathcal{L}_o$. Then, there exists a m.u. sequence $(s_n) \in \mathrm{p_o}^{-1}\mathrm{pre}\,\mathcal{L}_o$ such that $\lim(s_n) = w$ and $(\mathrm{p_o} s_n) \in \mathrm{pre}\,\mathcal{L}_o$. Case that $(\mathrm{p_o} s_n)$ is a bounded sequence, then $\lim(\mathrm{p_o} w_n) = \mathrm{p_o^\omega} w \in \mathrm{pre}\,\mathcal{L}_o$ and $w \in \mathrm{p_o}^{-\omega}\mathrm{pre}\,\mathcal{L}_o$. Otherwise, $\lim(\mathrm{p_o} s_n) \in \mathrm{clo}\,\mathcal{L}_o$ and $w \in \mathrm{p_o}^{-\omega}\mathrm{clo}\,\mathcal{L}_o$. $(\supseteq)$: (1) We pick $w \in \mathrm{p_o}^{-\omega}\mathrm{clo}\,\mathcal{L}_o$. Then, there exists a m.u. sequence $(\mathrm{p_o} s_n) \in \mathrm{pre}\,\mathcal{L}_o$ and $(s_n) \subseteq \mathrm{pre}\,\mathrm{p_o}^{-\omega}\mathcal{L}_o$. Thus, $w = \lim(s_n) \in \mathrm{clo}\,\mathrm{p_o}^{-\omega}\mathcal{L}_o$. (2) We pick $w \in \mathrm{p_o}^{-\omega}\mathrm{pre}\,\mathcal{L}_o$. Then, there exists $s \in \mathrm{pre}\,\mathcal{L}_o$ such that $\mathrm{p_o^\omega} w = s$, i.e. there exists a m.u. sequence $(s_n) \subseteq \mathrm{p_o}^{-1} s \subseteq \mathrm{p_o}^{-1}\mathrm{pre}\,\mathcal{L}_o = \mathrm{pre}\,\mathrm{p_o}^{-\omega}\mathcal{L}_o$ and $w = \lim(s_n) \in \mathrm{clo}\,\mathrm{p_o}^{-\omega}\mathcal{L}_o$. □

For ω-languages, we use the same definition of ω-controllability as in (Moor et al., 2011): given $\Sigma_{uc} \subseteq \Sigma$ and $\mathcal{L}, \mathcal{H} \subseteq \Sigma^\omega$. Then, $\mathcal{H}$ is said to be ω*-controllable* w.r.t. $(\Sigma_{uc}, \mathcal{L})$ if for all $s \in (\mathrm{pre}\,\mathcal{L}) \cap (\mathrm{pre}\,\mathcal{H})$ there exists a $\mathcal{V}_s \subseteq \mathcal{L} \cap \mathcal{H}$ with $s \in \mathrm{pre}\,\mathcal{V}_s$, and

(i) $\mathrm{pre}\,\mathcal{V}_s$ is controllable w.r.t. $\mathrm{pre}\,\mathcal{L}$, and
(ii) $\mathcal{V}_s$ is relatively topologically closed w.r.t. $\mathcal{L}$.

For $\mathcal{H} \subseteq \mathcal{L}$, our notion of ω-controllability is equivalent to ω-controllability as introduced by Thistle and Wonham (1994). Addressing situations where $\mathcal{H}$ is not necessarily a subset of $\mathcal{L}$, our notion of ω-controllability implies that $\mathcal{L}$ and $\mathcal{H}$ are non-conflicting. Furthermore, ω-controllability is preserved under arbitrary union, see (Moor et al., 2011). Given $\mathcal{L},\ \mathcal{K} \subseteq \Sigma^\omega$, and a set of observable events $\Sigma_o \subseteq \Sigma$, we say that $\mathcal{K}$ is ω*-normal w.r.t.* $(\Sigma_o, \mathcal{L})$, if $\mathcal{K} = (\mathrm{p_o}^{-\omega}\mathrm{p_o^\omega}\mathcal{K}) \cap \mathcal{L}$, see e.g. (Kumar et al., 1992).

Lemma 3. Let $\mathcal{K}$ be rel. closed w.r.t. $\mathcal{L} \subseteq \Sigma^\omega$ and $\Sigma_o \subseteq \Sigma$, then $\mathcal{K}$ is ω-normal w.r.t. $(\Sigma_o, \mathcal{L})$ if $\mathrm{pre}\,\mathcal{K}$ is normal w.r.t. $(\Sigma_o,\ \mathrm{pre}\,\mathcal{L})$.

Proof. $\mathcal{K} = (\mathrm{clo}\,\mathcal{K}) \cap \mathcal{L} = (\lim((\mathrm{p_o}^{-1}\mathrm{p_o}\mathrm{pre}\,\mathcal{K}) \cap (\mathrm{pre}\,\mathcal{L}))) \cap \mathcal{L} = (\lim\mathrm{pre}\,\mathrm{p_o}^{-\omega}\mathrm{p_o^\omega}\mathcal{K})\ \cap\ (\lim\mathrm{pre}\,\mathcal{L})\ \cap\ \mathcal{L} = (\mathrm{clo}\,\mathrm{p_o}^{-\omega}\mathrm{p_o^\omega}\mathcal{K})\ \cap\ \mathcal{L} \supseteq (\mathrm{p_o}^{-\omega}\mathrm{p_o^\omega}\mathcal{K})\ \cap\ \mathcal{L} \supseteq \mathcal{K}$. Equality implies that $\mathcal{K}$ is ω-normal. □

3. CLOSED-LOOP WITH EXTERNAL SIGNALS

The closed-loop configuration under consideration consists of a controller component, a plant component, and three ports for system interconnection; see Figure 1, on the left. The motivation of explicitly addressing external interaction is to specify the relationship between internal and external behaviour as a formal requirement for the controller design.

Each of the three ports is realized by synchronization of alternating input-events and output-events, from alphabets denoted U_{-} and Y_{-}, respectively. As in (Moor et al., 2011; Perk et al., 2006), this particular form of system interconnection refers to the notion of input-output systems from Willems (1991) and is a crucial prerequisite for our results on abstraction based controller design in Section 4.

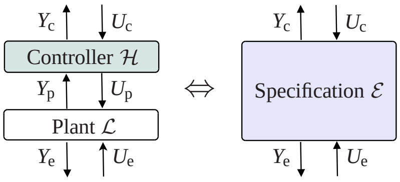

Fig. 1. Closed-loop configuration

Internally, the plant and the controller synchronize alternating symbols from $\Sigma_p = U_p \dot\cup Y_p$, and, thus, form a closed-loop configuration, similar to the common setting of sampled data continuous control systems. Furthermore, the controller interacts with a high-level operator, while the plant is synchronized with a low-level environment. We take the perspective that the operator seeks to affect the environment according to high-level commands from U_c. The controller is meant to implement each high-level command on the plant by applying suitable events from U_p, while monitoring the plant responses ranging in Y_p. Eventually, the controller shall provide a high-level feedback event from Y_c to the operator, in order to receive the subsequent high-level command. A specification referring to the overall alphabet is meant to relate high-level events from $\Sigma_c = U_c \dot\cup Y_c$ to low-level events from $\Sigma_e = U_e \dot\cup Y_e$, and, thereby, formally define the consequences of high-level commands; see also Figure 1, to the right.

For the further discussion, we summarize the relevant parameters as a *control problem* and subsequently introduce conditions and requirements to characterize acceptable *solutions*.

Definition 4. A ***control problem*** consists of

$\Sigma := \Sigma_p \dot\cup \Sigma_e \dot\cup \Sigma_c$, the overall alphabet,
$\Sigma_c := U_c \dot\cup Y_c$, the *high-level control events*,
$\Sigma_p := U_p \dot\cup Y_p$, the *internal plant events*,
$\Sigma_e := U_e \dot\cup Y_e$, the *environment events*,
$\mathcal{L} \subseteq (\Sigma_p \dot\cup \Sigma_e)^\omega$, the *plant behaviour*, and
$\mathcal{E} \subseteq \Sigma^\omega$, the *specification*.

Throughout this paper, the individual alphabets are obvious from the context and we concisely refer to the control problem by $(\Sigma, \mathcal{L}, \mathcal{E})$. Furthermore, we denote

$\Sigma_{pe} := \Sigma_p \dot\cup \Sigma_e$, the *plant alphabet*,
$\Sigma_{cp} := \Sigma_c \dot\cup \Sigma_p$, the *controller alphabet*,
$\Sigma_{ce} := \Sigma_c \dot\cup \Sigma_e$, the *external alphabet*,
$\Sigma_{uc} := U_c \dot\cup Y_p \dot\cup \Sigma_e$, the *uncontrollable events*, and
$\Sigma_o := \Sigma_c \dot\cup \Sigma_p$, the *observable events*.

Projections from strings or infinite strings over Σ, are denoted p_{-} and p_{-}^{ω}, respectively, with a subscript to indicate the respective range; e.g., p_{pe} for the projection from Σ^* to Σ_{pe}^*.

3.1 Plant properties

For the intended interpretation of inputs and outputs, the plant behaviour $\mathcal{L} \subseteq \Sigma_{pe}^\omega$ must exhibit alternating input and output events, and accept any input event from the controller and from the environment. For the acceptance of input events, we refer to the notion of a locally free input; see also (Perk et al., 2006).

Definition 5. For a language $L \subseteq \Sigma^*$, the alphabet $U \subseteq \Sigma$ is a *locally free input* if

$$(\forall s \in \Sigma^*, \mu, \mu' \in U)\,[\,s\mu \in \mathrm{pre}\,L \Rightarrow s\mu' \in \mathrm{pre}\,L\,]\,.$$

Formally, we require the plant behaviour to possess the below properties P1 and P2 and refer to $\mathcal{L}$ as an ***IO-plant***.

P1 $\mathcal{L} \subseteq ((Y_p U_p)^* (Y_e U_e)^*)^\omega \subseteq \Sigma_{pe}^\omega$.

P2 $\mathrm{pre}\,\mathcal{L}$ possesses locally free inputs U_p and U_e.

For the subsequent discussion, it turns out convenient to raise $\mathcal{L} \subseteq \Sigma_{pe}^\omega$ to the overall alphabet Σ, and to consider

$$\mathcal{L}_\Sigma := \mathrm{p}_{pe}^{-\omega}(\mathcal{L} \cup \mathrm{pre}\,\mathcal{L}) \cap \mathrm{clo}\,(\,(\,(Y_p (Y_c U_c)^* U_p)^* \,(Y_e U_e)^*\,)^\omega\,)$$

as the *full plant behaviour*. The particular construction ensures that the inverse projection does not introduce artificial liveness properties while enforcing the intended event order. Moreover, if $\mathcal{L}$ is an IO-plant, then $\mathcal{L}_\Sigma$ possesses locally free inputs U_c, $U_p \dot\cup Y_c$ and U_e by construction.

3.2 Specification properties

The main purpose of the language inclusion specification $\mathcal{E} \subseteq \Sigma^\omega$ is to relate external to internal signals. However, for the hierarchical control architecture in Section 4, we also require that the external closed-loop behaviour again possesses the plant properties P1 and P2. In particular, the external closed-loop must persistently provide high-level feedback ranging in Y_c and it must accept any external input events from U_c and U_e. Technically, the ***IO-specification*** $\mathcal{E}$ must satisfy E1 and E2:

E1 $\mathcal{E} \subseteq (\,(\,(Y_p U_p)^* \,(Y_e U_e)^*\,)^* \,(Y_p (Y_c U_c)^+ U_p)\,)^\omega$.

E2 $\mathrm{pre}\,\mathcal{E}$ possesses locally free inputs U_c and U_e.

3.3 Solution to the control problem

Given a control problem $(\Sigma, \mathcal{L}, \mathcal{E})$ with an IO-plant $\mathcal{L}$ and an IO-specification $\mathcal{E}$, consider a candidate controller $\mathcal{H} \subseteq \Sigma_{cp}^\omega$. For convenience, we write $\mathcal{H}_\Sigma := \mathrm{p}_{cp}^{-\omega}\mathcal{H} \subseteq \Sigma^\omega$ for the controller behaviour w.r.t. the overall alphabet. For $\mathcal{H}$ to be a ***solution*** to the control problem, it must enforce the specification and satisfy the controllability condition w.r.t. the plant behaviour. Formally, we impose the following conditions C1 and C2.

C1 $\mathcal{H}$ enforces the specification $\mathcal{E}$, i.e., $\mathcal{L}_\Sigma \cap \mathcal{H}_\Sigma \subseteq \mathcal{E}$.

C2 $\mathcal{H}_\Sigma$ is ω-controllable w.r.t. $(\Sigma_{uc}, \mathcal{L}_\Sigma)$.

Note that C2 implies that $\mathcal{L}_\Sigma$ and $\mathcal{H}_\Sigma$ are non-conflicting. Moreover, by C1 and E1, we obtain the *full closed-loop behaviour*

$$\mathcal{K} := \mathcal{L} \parallel \mathcal{H} := (\mathrm{p}_{pe}^{-\omega}\mathcal{L}) \cap (\mathrm{p}_{cp}^{-\omega}\mathcal{H}) = \mathcal{L}_\Sigma \cap \mathcal{H}_\Sigma\,.$$

Thus, the closed-loop interconnection of the plant $\mathcal{L}$ with the controller $\mathcal{H}$ is non-blocking.

3.4 Closed-loop properties

This subsection relates solutions of the control problem to properties of the full closed-loop behaviour.

Proposition 6. If $\mathcal{H}$ is a solution to the control problem $(\Sigma, \mathcal{L}, \mathcal{E})$, where $\mathcal{L}$ is an IO-plant, then the full closed-loop behaviour $\mathcal{K} = \mathcal{L}_\Sigma \cap \mathcal{H}_\Sigma$ satisfies K1–K5:

K1 $\mathcal{K}$ enforces the specification $\mathcal{E}$, i.e., $\mathcal{K} \subseteq \mathcal{E}$,
K2 $\mathcal{K}$ is ω-controllable w.r.t.$(\Sigma_{\mathrm{uc}}, \mathcal{L}_\Sigma)$,
K3 $\mathcal{K}$ is ω-normal w.r.t. $(\Sigma_{\mathrm{o}}, \mathcal{L}_\Sigma)$,
K4 $\mathrm{pre}\,\mathcal{K}$ is normal w.r.t. $(\Sigma_{\mathrm{o}}, \mathrm{pre}\,\mathcal{L}_\Sigma)$,
K5 $\mathrm{pre}\,\mathcal{K}$ possesses locally free inputs U_{c} and U_{e}.

Proof. K1 and K2 are immediate consequences of C1 and C2, respectively. Regarding K3 observe that

$$\mathcal{K} \subseteq (\mathrm{p}_{\mathrm{cp}}^{-\omega}\mathrm{p}_{\mathrm{cp}}^{\omega}\mathcal{K}) \cap \mathcal{L}_\Sigma = (\mathrm{p}_{\mathrm{cp}}^{-\omega}\mathrm{p}_{\mathrm{cp}}^{\omega}(\mathcal{H}_\Sigma \cap \mathcal{L}_\Sigma)) \cap \mathcal{L}_\Sigma \subseteq$$
$$(\mathrm{p}_{\mathrm{cp}}^{-\omega}\mathrm{p}_{\mathrm{cp}}^{\omega}\mathrm{p}_{\mathrm{cp}}^{-\omega}\mathcal{H}) \cap (\mathrm{p}_{\mathrm{cp}}^{-\omega}\mathrm{p}_{\mathrm{cp}}^{\omega}\mathcal{L}_\Sigma) \cap \mathcal{L}_\Sigma = \mathcal{H}_\Sigma \cap \mathcal{L}_\Sigma = \mathcal{K}.$$

K4 is obtained by

$$\mathrm{pre}\,\mathcal{K} \subseteq (\mathrm{p}_{\mathrm{cp}}^{-1}\mathrm{p}_{\mathrm{cp}}\mathrm{pre}\,\mathcal{K}) \cap (\mathrm{pre}\,\mathcal{L}_\Sigma) =$$
$$(\mathrm{p}_{\mathrm{cp}}^{-1}\mathrm{p}_{\mathrm{cp}}\mathrm{pre}\,(\mathcal{H}_\Sigma \cap \mathcal{L}_\Sigma)) \cap (\mathrm{pre}\,\mathcal{L}_\Sigma) \subseteq$$
$$(\mathrm{p}_{\mathrm{cp}}^{-1}\mathrm{p}_{\mathrm{cp}}\mathrm{pre}\,\mathcal{H}_\Sigma) \cap (\mathrm{p}_{\mathrm{cp}}^{-1}\mathrm{p}_{\mathrm{cp}}\mathrm{pre}\,\mathcal{L}_\Sigma) \cap (\mathrm{pre}\,\mathcal{L}_\Sigma) =$$
$$(\mathrm{pre}\,\mathcal{H}_\Sigma) \cap (\mathrm{pre}\,\mathcal{L}_\Sigma) = \mathrm{pre}\,(\mathcal{H}_\Sigma \cap \mathcal{L}_\Sigma) = \mathrm{pre}\,\mathcal{K}.$$

For the penultimate equality, recall that C2 implies that $\mathcal{L}_\Sigma$ and $\mathcal{H}_\Sigma$ are non-conflicting. Regarding K5, we pick $s,\ r \in \mathrm{pre}\,\mathcal{K}$, $\mu,\ \mu' \in U_{\mathrm{e}}$, and $\nu,\ \nu' \in U_{\mathrm{c}}$, such that $s\mu \in \mathrm{pre}\,\mathcal{K}$ and $r\nu \in \mathrm{pre}\,\mathcal{K}$. Observe that $s\mu,\ r\nu \in \mathrm{pre}\,\mathcal{K} \subseteq \mathrm{pre}\,\mathcal{L}_\Sigma$. According to P2 it follows that $s\mu' \in \mathrm{pre}\,\mathcal{L}_\Sigma$. Furthermore, the locally free input U_{c} of $\mathrm{pre}\,\mathcal{L}_\Sigma$ implies that $s\nu' \in \mathrm{pre}\,\mathcal{L}_\Sigma$. From ω-controllability of $\mathcal{H}_\Sigma$ w.r.t. $(\Sigma_{\mathrm{uc}}, \mathcal{L}_\Sigma)$ and $s,\ r \in \mathrm{pre}\,\mathcal{H}_\Sigma$ follows that $s\mu',\ r\nu' \in \mathrm{pre}\,\mathcal{H}_\Sigma$. Recall again that $\mathcal{L}_\Sigma$ and $\mathcal{H}_\Sigma$ are non-conflicting, to obtain $s\mu',\ r\nu' \in (\mathrm{pre}\,\mathcal{L}_\Sigma) \cap (\mathrm{pre}\,\mathcal{H}_\Sigma) = \mathrm{pre}\,\mathcal{K}$. □

3.5 Controller synthesis

Vice versa, any ω-language that satisfies properties K1–K4 can be shown to be a solution to the control problem.

Proposition 7. Given a control problem $(\Sigma, \mathcal{L}, \mathcal{E})$, consider any closed-loop candidate $\mathcal{K} \subseteq \Sigma^\omega$. If $\mathcal{K}$ satisfies K1–K4, then, the controller $\mathcal{H} = \mathrm{p}_{\mathrm{cp}}^{\omega}\mathcal{K}$ solves $(\Sigma, \mathcal{L}, \mathcal{E})$.

Proof. According to K1 and K3, we have that $\mathcal{L}_\Sigma \cap \mathcal{H}_\Sigma = \mathcal{L}_\Sigma \cap (\mathrm{p}_{\mathrm{cp}}^{-\omega}\mathrm{p}_{\mathrm{cp}}^{\omega}\mathcal{K}) = \mathcal{K} \subseteq \mathcal{E}$ and $\mathcal{H}$ satisfies C1. To establish C2, pick an arbitrary $s \in (\mathrm{pre}\,\mathcal{L}_\Sigma) \cap (\mathrm{pre}\,\mathrm{p}_{\mathrm{cp}}^{-\omega}\mathrm{p}_{\mathrm{cp}}^{\omega}\mathcal{K})$. By K4, we obtain $s \in \mathrm{pre}\,\mathcal{K}$. According to K2, we can choose $\mathcal{V}_s \subseteq \mathcal{L}_\Sigma \cap \mathcal{K} \subseteq \mathcal{L}_\Sigma \cap \mathcal{H}_\Sigma$ such that $s \in \mathrm{pre}\,\mathcal{V}_s$, and $\mathrm{pre}\,\mathcal{V}_s$ is controllable w.r.t. $(\Sigma_{\mathrm{uc}}, \mathrm{pre}\,\mathcal{L}_\Sigma)$, and, $\mathcal{V}_s$ is relatively closed w.r.t. $\mathcal{L}_\Sigma$. Hence, $\mathcal{H}_\Sigma$ is ω-controllable and satisfies C2. □

As a consequence of the above two propositions and in compliance to common approaches in supervisory control, solutions to a control problem can be obtained from the supremal closed-loop behaviour, as characterized by K1–K4.

If $\mathcal{L}$ and $\mathcal{E}$ can be represented by limits of regular $*$-languages, and, if $\mathcal{E}$ is relatively topologically closed w.r.t. $\mathcal{L}$, then, the results presented by (Moor et al., 2012) can be utilized to obtain a practical solution to the control problem $(\Sigma, \mathcal{L}, \mathcal{E})$. Referring to the discussion in Section 6, (Moor et al., 2012), the following proposition gives a representation of the supremal solution $\mathcal{H}^{\Uparrow}$ of the control problem $(\Sigma, \mathcal{L}, \mathcal{E})$:

Proposition 8. Given a control problem $(\Sigma, \mathcal{L}, \mathcal{E})$, represented by $\mathcal{L}_\Sigma = \lim L_\Sigma$, $\mathcal{E} = \lim E$, where $L_\Sigma, E \subseteq \Sigma^*$ are complete, $\mathcal{E} = (\mathrm{clo}\,\mathcal{E}) \cap \mathcal{L}$, and $E = (\mathrm{pre}\,\mathcal{E}) \cap L$, for any $K \subseteq \Sigma^*$ that satisfies the requirements (L1)–(L5) given in (Moor et al., 2012), the controller $\mathcal{H} = \mathrm{p}_{\mathrm{cp}}^{\omega} \lim K$ solves $(\Sigma, \mathcal{L}, \mathcal{E})$. For $\mathcal{H}^{\uparrow} := \mathrm{p}_{\mathrm{cp}}^{\omega} \lim K^{\uparrow}$, where $K^{\uparrow}$ denotes the supremal solution w.r.t. (L1)–(L5), we have $\mathcal{L} \parallel \mathcal{H}^{\uparrow} = \mathcal{L} \parallel \mathcal{H}^{\Uparrow}$.

Proof. To prove the first part, we show, that $\mathcal{K} = \lim K$ satisfies K1–K4. Regarding K1, observe that by L4 $\lim K \subseteq \lim E = \lim((\mathrm{pre}\,\mathcal{E}) \cap L_\Sigma) = (\mathrm{clo}\,\mathcal{E}) \cap \mathcal{L}_\Sigma = \mathcal{E}$. Regarding K2, we pick $\mathcal{K}$ as candidate $\mathcal{V}_s$ and observe that controllability of $\mathrm{pre}\,\mathcal{K}$ follows from L2. Relative closedness is given by L5, since $\lim K = \lim(\mathrm{pre}\,K \cap L_\Sigma) = (\mathrm{clo}\,\mathcal{K}) \cap \mathcal{L}_\Sigma$. Thus, K2 is satisfied. Regarding K3 and K4, observe, that normality of $\mathrm{pre}\,\mathcal{K}$ w.r.t. $(\Sigma_{\mathrm{o}}, \mathrm{pre}\,\mathcal{L}_\Sigma)$ follows from L3. Normality of $\mathcal{K}$ is given by relative closedness of $\mathcal{K}$ w.r.t. $\mathcal{L}_\Sigma$ and K4. By Proposition 7, $\mathcal{H} = \mathrm{p}_{\mathrm{cp}}^{\omega} \lim K$ solves $(\Sigma, \mathcal{L}, \mathcal{E})$. Regarding supremality, note that $\mathcal{L}_\Sigma \parallel \mathcal{H}^{\uparrow} = \mathcal{L}_\Sigma \cap (\mathrm{p}_{\mathrm{cp}}^{-\omega} \lim \mathcal{K}^{\Uparrow}) \subseteq \mathcal{L}_\Sigma \parallel \mathcal{H}^{\Uparrow}$, since $\mathcal{H}^{\uparrow}$ solves $(\Sigma, \mathcal{L}, \mathcal{E})$. We prove the reverse direction by $\mathcal{L} \parallel \mathcal{H} \subseteq \mathcal{L}_\Sigma \cap (\mathrm{clo}\,(\mathcal{L} \parallel \mathcal{H})) = \mathcal{L}_\Sigma \cap \mathrm{clo}\,\lim K = \lim L_\Sigma \cap \lim \mathrm{pre}\,K = \lim(L_\Sigma \cap \mathrm{pre}\,K) = \lim K$. Hence, $\lim K^{\uparrow} = \mathcal{L} \parallel \mathcal{H}^{\uparrow} \subseteq \mathcal{L} \parallel \mathcal{H}^{\Uparrow} \subseteq \lim K^{\uparrow}$, and $\mathcal{L} \parallel \mathcal{H}^{\uparrow} = \mathcal{L} \parallel \mathcal{H}^{\Uparrow}$. □

Future work focuses on the development of algorithms for the case of more general specifications, which is still an open question. See also (Thistle and Wonham, 1994; Thistle and Lamouchi, 2009) and the literature cited therein.

4. HIERARCHICAL CONTROLLER DESIGN

Consider a control problem $(\Sigma, \mathcal{L}, \mathcal{E})$, a solution $\mathcal{H}$ and the full closed-loop behaviour $\mathcal{K} = \mathcal{L}_\Sigma \cap \mathcal{H}_\Sigma$. The *external closed-loop behaviour* $\mathcal{L}^{\mathrm{H}} := \mathrm{p}_{\mathrm{ce}}^{\omega}\mathcal{K}$ can again be interpreted as a plant. Thus, given a specification $\mathcal{E}^{\mathrm{H}}$, we obtain another control problem $(\Sigma^{\mathrm{H}}, \mathcal{L}^{\mathrm{H}}, \mathcal{E}^{\mathrm{H}})$. Provided we find a solution $\mathcal{H}^{\mathrm{H}}$, we end up with a hierarchical control architecture; see Figure 2, to the right.

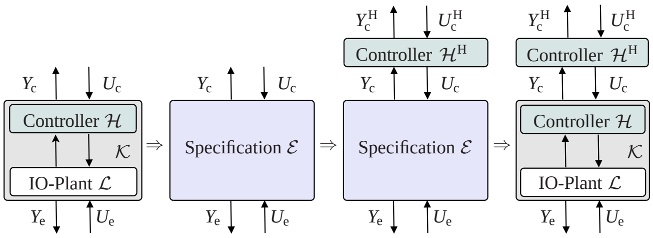

Fig. 2. Abstraction based hierarchical controller design

Rather than to solve $(\Sigma^{\mathrm{H}}, \mathcal{L}^{\mathrm{H}}, \mathcal{E}^{\mathrm{H}})$ directly, we propose to use the specification $\mathrm{p}_{\mathrm{ce}}^{\omega}\mathcal{E}$ as an abstraction of the plant behaviour $\mathcal{L}^{\mathrm{H}}$; see again Figure 2. Since $\mathcal{K} \subseteq \mathcal{E}$ implies $\mathcal{L}^{\mathrm{H}} = \mathrm{p}_{\mathrm{ce}}^{\omega}\mathcal{K} \subseteq \mathrm{p}_{\mathrm{ce}}^{\omega}\mathcal{E}$, solutions $\mathcal{H}^{\mathrm{H}}$ of $(\Sigma^{\mathrm{H}}, \mathrm{p}_{\mathrm{ce}}^{\omega}\mathcal{E}, \mathcal{E}^{\mathrm{H}})$ are readily observed to also satisfy C1 for the actual control problem $(\Sigma^{\mathrm{H}}, \mathcal{L}^{\mathrm{H}}, \mathcal{E}^{\mathrm{H}})$. In contrast to the actual closed-loop $\mathcal{K}$, the specification $\mathcal{E}$ does not express how the control objective is achieved and, hence, is expected to be considerably less complex.

However, the proposed approach raises two questions.

- Are the plant properties P1 and P2 of $\mathcal{L}$ retained under closed-loop composition and, thus, also satisfied by $\mathcal{L}^{\mathrm{H}}$?
- Can we guarantee that the solutions of $(\Sigma^{\mathrm{H}}, \mathrm{p}_{\mathrm{ce}}^{\omega}\mathcal{E}, \mathcal{E}^{\mathrm{H}})$ also solve the actual problem $(\Sigma^{\mathrm{H}}, \mathcal{L}^{\mathrm{H}}, \mathcal{E}^{\mathrm{H}})$, i.e., possess both properties C1 and C2?

We provide affirmative answers to both questions.

4.1 Non-anticipating IO-plant

In (Moor et al., 2011), it has been shown that locally free inputs, as imposed by P2, do in general not imply a non-blocking closed-loop for an abstraction based controller design. More specifically, the cited paper elaborates a variation of Willems' notion of non-anticipating input-output systems as a sufficient structural plant property for a non-blocking closed-loop. Based on these considerations, we impose the additional requirement P3 on $\mathcal{L}$ and refer to the plant as a ***non-anticipating IO-plant***.

P3 $\mathcal{L}$ is ω-controllable w.r.t. $(U_\mathrm{p} \dot\cup U_\mathrm{e}, \mathrm{clo}\,\mathcal{L})$.

While P2 requires the plant to accept any input locally, P3 requires that the liveness properties possessed by the plant may at no instance of time restrict the input in its infinite future; see Moor et al. (2011) for a detailed discussion of P3, including examples. The following two propositions draw conclusions from P3 regarding the full plant behaviour and the closed-loop behaviour, respectively.

Proposition 9. If $\mathcal{L}$ is a non-anticipating IO-plant, then $\mathcal{L}_\Sigma$ is ω-controllable w.r.t. $(\Sigma_\mathrm{c} \dot\cup U_\mathrm{p} \dot\cup U_\mathrm{e}, \mathrm{clo}\,\mathcal{L}_\Sigma)$.

Proof. From the definition of $\mathcal{L}_\Sigma$, we note that $\mathrm{pre}\,\mathcal{L}_\Sigma \subseteq \mathrm{pre}\,(\mathrm{p}_\mathrm{pe}^{-\omega}(\mathcal{L} \cup \mathrm{pre}\,\mathcal{L})) = \mathrm{p}_\mathrm{pe}^{-1}\mathrm{pre}\,\mathcal{L}$. Pick an arbitrary string $s \in \mathrm{pre}\,\mathcal{L}_\Sigma$, let $r := \mathrm{p}_\mathrm{pe}s$, and observe that $r \in \mathrm{pre}\,\mathcal{L}$. Since $\mathcal{L}$ is non-anticipating, we can choose $\hat{\mathcal{V}}_r \subseteq \mathcal{L}$, such that $r \in \mathrm{pre}\,\hat{\mathcal{V}}_r$, and $\mathrm{pre}\,\hat{\mathcal{V}}_r$ is controllable w.r.t. $(U_\mathrm{p} \dot\cup U_\mathrm{e}, \mathrm{pre}\,\mathcal{L})$, and $\hat{\mathcal{V}}_r$ is relatively closed w.r.t. $\mathrm{clo}\,\mathcal{L}$. Recall that relative closedness w.r.t. a closed language implies closedness. In particular, $\hat{\mathcal{V}}_r$ is closed. To establish the non-anticipating property of $\mathcal{L}_\Sigma$, consider the candidate

$$\mathcal{V}_s := (\mathrm{p}_\mathrm{pe}^{-\omega}(\hat{\mathcal{V}}_r \cup \mathrm{pre}\,\hat{\mathcal{V}}_r)) \,\cap\, (\mathrm{clo}\,((Y_\mathrm{p}(Y_\mathrm{c}U_\mathrm{c})^*U_\mathrm{p})^*(Y_\mathrm{e}U_\mathrm{e})^*)^\omega).$$

Clearly, $\mathcal{V}_s \subseteq \mathcal{L}_\Sigma$ and $\mathrm{pre}\,\mathcal{V}_s \subseteq \mathrm{pre}\,\mathrm{p}_\mathrm{pe}^{-\omega}(\hat{\mathcal{V}}_r \cup \mathrm{pre}\,\hat{\mathcal{V}}_r) = \mathrm{p}_\mathrm{pe}^{-1}\mathrm{pre}\,\hat{\mathcal{V}}_r$. Further, we have that $s \in \mathrm{pre}\,\mathcal{V}_s$, since $\mathrm{p}_\mathrm{pe}s = r \subseteq \mathrm{pre}\,\hat{\mathcal{V}}_r$ and $s \subseteq \mathrm{pre}\,((Y_\mathrm{p}(Y_\mathrm{c}U_\mathrm{c})^*U_\mathrm{p})^*(Y_\mathrm{e}U_\mathrm{e})^*)^\omega$. To show controllability, pick an arbitrary string $\hat{s} \in \mathrm{pre}\,\mathcal{V}_s$ and $\sigma \in \Sigma_\mathrm{c} \dot\cup U_\mathrm{p} \dot\cup U_\mathrm{e}$ such that $\hat{s}\sigma \in \mathrm{pre}\,\mathcal{L}_\Sigma$. In particular, $\mathrm{p}_\mathrm{pe}\hat{s} \in \mathrm{p}_\mathrm{pe}\mathrm{p}_\mathrm{pe}^{-1}\mathrm{pre}\,\hat{\mathcal{V}}_r = \mathrm{pre}\,\hat{\mathcal{V}}_r$ and $\mathrm{p}_\mathrm{pe}(\hat{s}\sigma) \in \mathrm{p}_\mathrm{pe}\mathrm{pre}\,\mathcal{L}_\Sigma \subseteq \mathrm{pre}\,\mathcal{L}$. Controllability of $\mathrm{pre}\,\hat{\mathcal{V}}_r$ w.r.t. $\mathrm{pre}\,\mathcal{L}$ implies that $\mathrm{p}_\mathrm{pe}(\hat{s}\sigma) \in \mathrm{pre}\,\hat{\mathcal{V}}_r$. In addition, there exists $u \in \Sigma_\mathrm{pe}^\omega$, such that $\mathrm{p}_\mathrm{pe}(\hat{s}\sigma)u \in \hat{\mathcal{V}}_r$. We choose $w \in \Sigma^\omega$ such that $\hat{s}\sigma w \in \mathrm{clo}\,((Y_\mathrm{p}(Y_\mathrm{c}U_\mathrm{c})^*U_\mathrm{p})^*(Y_\mathrm{e}U_\mathrm{e})^*)^\omega$ and $\mathrm{p}_\mathrm{pe}^\omega(\hat{s}\sigma w) = \mathrm{p}_\mathrm{pe}(\hat{s}\sigma)u$. Note that $\hat{s}\sigma w \in \mathcal{V}_s$ and, hence, $\hat{s}\sigma \in \mathrm{pre}\,\mathcal{V}_s$. Finally, observe that

$$\begin{aligned}\mathcal{V}_s &= (\mathrm{p}_\mathrm{pe}^{-\omega}(\hat{\mathcal{V}}_r \cup \mathrm{pre}\,\hat{\mathcal{V}}_r)) \,\cap\, (\mathrm{clo}\,((Y_\mathrm{p}(Y_\mathrm{c}U_\mathrm{c})^*U_\mathrm{p})^*(Y_\mathrm{e}U_\mathrm{e})^*)^\omega)\\ &= (\mathrm{p}_\mathrm{pe}^{-\omega}\mathrm{clo}\,\hat{\mathcal{V}}_r \cup \mathrm{p}_\mathrm{pe}^{-\omega}\mathrm{pre}\,\hat{\mathcal{V}}_r) \cap (\mathrm{clo}\,((Y_\mathrm{p}(Y_\mathrm{c}U_\mathrm{c})^*U_\mathrm{p})^*(Y_\mathrm{e}U_\mathrm{e})^*)^\omega)\\ &= (\mathrm{clo}\,\mathrm{p}_\mathrm{pe}^{-\omega}\hat{\mathcal{V}}_r) \,\cap\, (\mathrm{clo}\,((Y_\mathrm{p}(Y_\mathrm{c}U_\mathrm{c})^*U_\mathrm{p})^*(Y_\mathrm{e}U_\mathrm{e})^*)^\omega).\end{aligned}$$

As the intersection of two closed languages, $\mathcal{V}_s$ is closed. □

Proposition 10. If $\mathcal{H}$ is a solution to the control problem $(\Sigma, \mathcal{L}, \mathcal{E})$, and if $\mathcal{L}$ is a non-anticipating IO-plant, then

K6 $\mathcal{K}$ is ω-controllable w.r.t. $(U_\mathrm{c} \dot\cup U_\mathrm{e}, \mathrm{clo}\,\mathcal{K})$.

Proof. We prove the claim by construction of a suitable $\mathcal{V}_s \subseteq \mathcal{K}$ for an arbitrarily chosen $s \in \mathrm{pre}\,\mathcal{K}$. Referring to Proposition 9, there exists $\tilde{\mathcal{V}}_s \subseteq \mathcal{L}_\Sigma$, such that $s \in \mathrm{pre}\,\tilde{\mathcal{V}}_s$, and $\mathrm{pre}\,\tilde{\mathcal{V}}_s$ is controllable w.r.t. $(\Sigma_\mathrm{c} \dot\cup U_\mathrm{p} \dot\cup U_\mathrm{e}, \mathrm{pre}\,\mathcal{L}_\Sigma)$, and $\tilde{\mathcal{V}}_s$ is relatively closed w.r.t. $\mathrm{clo}\,\mathcal{L}_\Sigma$. In particular, $\tilde{\mathcal{V}}_s$ is closed. By Proposition 6, $\mathcal{K}$ satisfies K1–K5. Referring to K2, we choose $\hat{\mathcal{V}}_s \subseteq \mathcal{K}$ with $s \in \mathrm{pre}\,\hat{\mathcal{V}}_s$, and $\mathrm{pre}\,\hat{\mathcal{V}}_s$ is controllable w.r.t. $(\Sigma_\mathrm{uc}, \mathrm{pre}\,\mathcal{L}_\Sigma)$, and $\hat{\mathcal{V}}_s$ is relatively closed w.r.t. $\mathcal{L}_\Sigma$.

To establish ω-controllability of $\mathcal{K}$ w.r.t. $\mathrm{clo}\,\mathcal{K}$, consider the candidate $\mathcal{V}_s := \tilde{\mathcal{V}}_s \cap \hat{\mathcal{V}}_s$. Clearly, $\mathcal{V}_s \subseteq \mathcal{K}$. Furthermore, $\mathcal{V}_s = \tilde{\mathcal{V}}_s \cap \hat{\mathcal{V}}_s = \tilde{\mathcal{V}}_s \cap (\mathrm{clo}\,\hat{\mathcal{V}}_s) \cap \mathcal{L}_\Sigma = \tilde{\mathcal{V}}_s \cap (\mathrm{clo}\,\hat{\mathcal{V}}_s) = (\mathrm{clo}\,\tilde{\mathcal{V}}_s) \cap (\mathrm{clo}\,\hat{\mathcal{V}}_s) \supseteq \mathrm{clo}\,\mathcal{V}_s$, i.e., $\mathcal{V}_s$ is closed and, thus, relatively closed w.r.t. any superset. To show controllability of $\mathrm{pre}\,\mathcal{V}_s$ w.r.t. $\mathrm{pre}\,\mathcal{K}$, we pick $r \in \mathrm{pre}\,(\tilde{\mathcal{V}}_s \cap \hat{\mathcal{V}}_s) \subseteq (\mathrm{pre}\,\tilde{\mathcal{V}}_s) \cap (\mathrm{pre}\,\hat{\mathcal{V}}_s)$ and $\sigma \in U_\mathrm{c} \dot\cup U_\mathrm{e}$ such that $r\sigma \in \mathrm{pre}\,\mathcal{K} \subseteq \mathrm{pre}\,\mathcal{L}_\Sigma$. By controllability of $\mathrm{pre}\,\tilde{\mathcal{V}}_s$ and $\mathrm{pre}\,\hat{\mathcal{V}}_s$, it follows that $r\sigma \in (\mathrm{pre}\,\tilde{\mathcal{V}}_s) \cap (\mathrm{pre}\,\hat{\mathcal{V}}_s)$. To establish $r\sigma \in \mathrm{pre}\,(\tilde{\mathcal{V}}_s \cap \hat{\mathcal{V}}_s)$, observe that each event in Σ is uncontrollable for either $\mathrm{pre}\,\tilde{\mathcal{V}}_s$ or $\mathrm{pre}\,\hat{\mathcal{V}}_s$. Thus, starting with $r_0 = r\sigma$, we can construct a strictly increasing sequence $(r_n) \subseteq (\mathrm{pre}\,\tilde{\mathcal{V}}_s) \cap (\mathrm{pre}\,\hat{\mathcal{V}}_s)$ with limit $w := \lim(r_n) \in (\mathrm{clo}\,\tilde{\mathcal{V}}_s) \cap (\mathrm{clo}\,\hat{\mathcal{V}}_s)$. Since $\tilde{\mathcal{V}}_s$ is closed, we have $w \in \tilde{\mathcal{V}}_s \subseteq \mathcal{L}_\Sigma$. By relative closedness of $\hat{\mathcal{V}}_s$ w.r.t. $\mathcal{L}_\Sigma$, we obtain $w \in \hat{\mathcal{V}}_s$. Hence, $r\sigma \in \mathrm{pre}\,(\tilde{\mathcal{V}}_s \cap \hat{\mathcal{V}}_s)$. □

4.2 Propagation of plant properties

We are now in the position to show that the plant properties P1–P3 are retained under closed-loop composition, i.e., the external closed-loop behaviour is again a non-anticipating IO-plant.

Theorem 11. For a non-anticipating IO-plant $\mathcal{L}$ and an IO-specification $\mathcal{E}$, consider a solution $\mathcal{H}$ of the control problem $(\Sigma, \mathcal{L}, \mathcal{E})$. Then the external closed-loop $\mathrm{p}_\mathrm{ce}^\omega\mathcal{K}$, with $\mathcal{K} = \mathcal{L}_\Sigma \cap \mathcal{H}_\Sigma$, is a non-anticipating IO-plant, too.

Proof. Regarding P1, we refer to K1 and E1 to obtain $\mathrm{p}_\mathrm{ce}^\omega\mathcal{K} \subseteq \mathrm{p}_\mathrm{ce}^\omega\mathcal{E} \subseteq ((Y_\mathrm{c}U_\mathrm{c})^*(Y_\mathrm{e}U_\mathrm{e})^*)^\omega$. Regarding P2, recall from K5 that $\mathcal{K}$ has locally free inputs U_c and U_e, that are preserved under projection to Σ_ce. We are left to show P3. Pick $s \in \mathrm{pre}\,\mathrm{p}_\mathrm{ce}^\omega\mathcal{K}$. Then, there exists $t \in \mathrm{pre}\,\mathcal{K}$ such that $\mathrm{p}_\mathrm{ce}t = s$. According to K6, we can choose $\hat{\mathcal{V}}_t \subseteq \mathcal{K}$ such that $\hat{t} \in \mathrm{pre}\,\hat{\mathcal{V}}_t$, and $\mathrm{pre}\,\hat{\mathcal{V}}_t$ is controllable w.r.t. $(U_\mathrm{c} \dot\cup U_\mathrm{e}, \mathrm{pre}\,\mathcal{K})$, and $\hat{\mathcal{V}}_t$ is closed. As a candidate to establish P3, let $\mathcal{V}_s := \mathrm{p}_\mathrm{ce}^\omega\hat{\mathcal{V}}_t$. Note that $\mathcal{V}_s \subseteq \mathrm{p}_\mathrm{ce}^\omega\mathcal{K}$. Further, $s = \mathrm{p}_\mathrm{ce}t \in \mathrm{p}_\mathrm{ce}\mathrm{pre}\,\hat{\mathcal{V}}_t = \mathrm{pre}\,\mathcal{V}_s$. To verify controllability of $\mathrm{pre}\,\mathcal{V}_s$, consider an arbitrary $\hat{s} \in \mathrm{pre}\,\mathcal{V}_s$ and $\sigma \in U_\mathrm{c} \dot\cup U_\mathrm{e}$ such that $\hat{s}\sigma \in \mathrm{pre}\,\mathrm{p}_\mathrm{ce}^\omega\mathcal{K}$. Then, there exists $\hat{t} \in \mathrm{pre}\,\mathcal{K}$ such that $\mathrm{p}_\mathrm{ce}\hat{t} = \hat{s}$ and $\hat{t} \in \mathrm{pre}\,\hat{\mathcal{V}}_t$. Furthermore, $\hat{t}\sigma \in \mathrm{pre}\,\mathcal{K}$, since $\hat{s}\sigma = \mathrm{p}_\mathrm{ce}(\hat{t}\sigma) \in \mathrm{p}_\mathrm{ce}\mathrm{pre}\,\mathcal{K}$. Controllability of $\mathrm{pre}\,\hat{\mathcal{V}}_t$ implies that $\hat{t}\sigma \in \mathrm{pre}\,\hat{\mathcal{V}}_t$ and $\mathrm{p}_\mathrm{ce}(\hat{t}\sigma) \in \mathrm{p}_\mathrm{ce}\mathrm{pre}\,\hat{\mathcal{V}}_t = \mathrm{pre}\,\mathcal{V}_s$. Thus, $\mathrm{pre}\,\mathcal{V}_s$ is controllable w.r.t. $(U_\mathrm{c} \dot\cup U_\mathrm{e}, \mathrm{pre}\,\mathrm{p}_\mathrm{ce}^\omega\mathcal{K})$. To verify closedness of $\mathcal{V}_s$, observe that $\mathcal{V}_s = \mathrm{p}_\mathrm{ce}^\omega\hat{\mathcal{V}}_t = (\mathrm{p}_\mathrm{ce}^\omega\mathrm{clo}\,\hat{\mathcal{V}}_t) \cap \Sigma_\mathrm{ce}^\omega = \mathrm{clo}\,\mathrm{p}_\mathrm{ce}^\omega\hat{\mathcal{V}}_t$. □

4.3 Abstraction based controller design

We adapt the argument presented in (Moor et al., 2011) to the closed-loop configuration with external signals.

Theorem 12. Given a control problem $(\Sigma, \mathcal{L}, \mathcal{E})$ with a non-anticipating IO-plant $\mathcal{L}$, let $\mathcal{L}' \subseteq \Sigma^\omega$ denote a plant abstraction, i.e., $\mathcal{L} \subseteq \mathcal{L}'$. Then, any solution of $(\Sigma, \mathcal{L}', \mathcal{E})$ solves $(\Sigma, \mathcal{L}, \mathcal{E})$.

To prove Theorem 12, we use the following technical lemma.

Lemma 13. For a non-anticipating IO-plant $\mathcal{L}$, the full behaviour can be represented as a union $\mathcal{L}_\Sigma = \cup_{a \in A}\mathcal{L}_a$, where for all $a \in A$

(i) $\mathcal{L}_a$ has locally free inputs U_c, $U_\mathrm{p} \dot\cup Y_\mathrm{c}$, and U_e.

(ii) $\mathcal{L}_a$ is closed.

Proof. Technically, P3 together with Proposition 9 implies that $\mathcal{L}_\Sigma$ itself is the supremal ω-controllable sublanguage of $\mathcal{L}_\Sigma$. Thus, by (Moor et al., 2011), Proposition 12, $\mathcal{L}_\Sigma$ can be represented as a union $\mathcal{L}_\Sigma = \cup_{a\in A}\mathcal{L}_a$, where, for each, $a \in A$, $\operatorname{pre}\mathcal{L}_a$ is controllable w.r.t. $(\Sigma_c \dot\cup U_p \dot\cup U_e, \operatorname{pre}\mathcal{L}_\Sigma)$ and $\mathcal{L}_a$ is closed. To establish (i), we pick $s \in \Sigma^*$, $\mu, \mu' \in U_c$, with $s\mu \in \operatorname{pre}\mathcal{L}_a$. The locally free input of $\operatorname{pre}\mathcal{L}_\Sigma$ implies $s\mu' \in \operatorname{pre}\mathcal{L}_\Sigma$, and controllability of $\operatorname{pre}\mathcal{L}_a$ w.r.t. $\operatorname{pre}\mathcal{L}_\Sigma$ implies, that $s\mu' \in \operatorname{pre}\mathcal{L}_a$. Locally free inputs $U_p \dot\cup Y_c$, and U_e are verified likewise.

Lemma 14. Under the hypothesis of Theorem 12, consider any solution $\mathcal{H}$ of the control problem $(\Sigma, \mathcal{L}', \mathcal{E})$. If $\mathcal{V}' \subseteq \mathcal{L}'_\Sigma \cap \mathcal{H}_\Sigma$, and if $\operatorname{pre}\mathcal{V}'$ is controllable w.r.t. $(\Sigma_{uc}, \mathcal{L}'_\Sigma)$, and if $\mathcal{V}'$ is relatively closed w.r.t. $\mathcal{L}'_\Sigma$, then $\mathcal{L}_\Sigma$ and $\mathcal{V}'$ are non-conflicting.

Proof. Pick an arbitrary string $s \in (\operatorname{pre}\mathcal{L}_\Sigma) \cap (\operatorname{pre}\mathcal{V}')$. Referring to Lemma 13, we represent $\mathcal{L}_\Sigma$ as $\mathcal{L}_\Sigma = \cup_{a\in A}\mathcal{L}_a$ with $\mathcal{L}_a$ satisfying conditions (i) and (ii). In particular, there exists $a \in A$ with $s \in \operatorname{pre}\mathcal{L}_a \subseteq \mathcal{L}_\Sigma \subseteq \mathcal{L}'_\Sigma$. To extend $s \in (\operatorname{pre}\mathcal{L}_a) \cap (\operatorname{pre}\mathcal{V}')$ by one event, pick σ such that $s\sigma \in \operatorname{pre}\mathcal{L}_a$. If $\sigma \in \Sigma_{uc}$, then controllability of $\operatorname{pre}\mathcal{V}'$ implies $s\sigma \in \operatorname{pre}\mathcal{V}'$, and we end up with $s\sigma \in (\operatorname{pre}\mathcal{L}_a) \cap (\operatorname{pre}\mathcal{V}')$. If, on the other hand, $\sigma \in U_p \dot\cup Y_c$, we obtain by Lemma 13, condition (ii), that $s(U_p \dot\cup Y_c) \subseteq \operatorname{pre}\mathcal{L}_a$. Referring to the event ordering in the definition of $\mathcal{L}_\Sigma$, we decompose $s = r\nu$ with $\nu \in U_c \dot\cup Y_p$. Again by the definition of $\mathcal{L}_\Sigma$, now using $r\nu \in \operatorname{pre}\mathcal{V}' \subseteq \operatorname{pre}\mathcal{L}_\Sigma$, we obtain the existence of $\sigma \in U_p \dot\cup Y_c$ such that $s\sigma \in \operatorname{pre}\mathcal{V}'$ and, thus, conclude with $s\sigma \in (\operatorname{pre}\mathcal{L}_a) \cap (\operatorname{pre}\mathcal{V}')$. Repeatedly extending s, we construct a strictly monotone sequence $(s_n) \subseteq (\operatorname{pre}\mathcal{L}_a) \cap (\operatorname{pre}\mathcal{V}')$ with limit $w := \lim(s_n) \in (\operatorname{clo}\mathcal{L}_a) \cap (\operatorname{clo}\mathcal{V}')$ and $s = s_0 < w$. Since $\mathcal{L}_a$ is closed, we obtain $w \in \mathcal{L}_a$ to observe $w \in \mathcal{L}_a \cap (\operatorname{clo}\mathcal{V}') \subseteq \mathcal{L}_\Sigma \cap \mathcal{L}'_\Sigma \cap (\operatorname{clo}\mathcal{V}') = \mathcal{L}_\Sigma \cap \mathcal{V}'$. In particular, $s \in \operatorname{pre}(\mathcal{L}_\Sigma \cap \mathcal{V}')$. □

Coming back to the proof of Theorem 12, note that $\mathcal{H}$ trivially satisfies C1, since $\mathcal{L}_\Sigma \cap \mathcal{H}_\Sigma \subseteq \mathcal{L}'_\Sigma \cap \mathcal{H}_\Sigma \subseteq \mathcal{E}$. Regarding C2, we pick any $s \in (\operatorname{pre}\mathcal{L}_\Sigma) \cap (\operatorname{pre}\mathcal{H}_\Sigma)$. Since $\mathcal{H}$ is a solution to $(\Sigma, \mathcal{L}', \mathcal{E})$, we can choose $\mathcal{V}'_s \subseteq \mathcal{L}'_\Sigma \cap \mathcal{H}_\Sigma$ such that $s \in \operatorname{pre}\mathcal{V}'_s$, and $\operatorname{pre}\mathcal{V}'_s$ is controllable w.r.t. $(\Sigma_{uc}, \operatorname{pre}\mathcal{L}'_\Sigma)$, and $\mathcal{V}'_s$ is relatively closed w.r.t. $\mathcal{L}'_\Sigma$. We choose the candidate $\mathcal{V}_s := \mathcal{V}'_s \cap \mathcal{L}_\Sigma$. Observe that $\mathcal{V}_s \subseteq \mathcal{L}'_\Sigma \cap \mathcal{H}_\Sigma \cap \mathcal{L}_\Sigma = \mathcal{H}_\Sigma \cap \mathcal{L}_\Sigma$ and $s \in (\operatorname{pre}\mathcal{L}_\Sigma) \cap (\operatorname{pre}\mathcal{H}_\Sigma) \cap (\operatorname{pre}\mathcal{V}'_s)$. By Lemma 14, we obtain $s \in \operatorname{pre}(\mathcal{L}_\Sigma \cap \mathcal{V}'_s) = \operatorname{pre}\mathcal{V}_s$. Regarding controllability, pick any $s\nu \in \operatorname{pre}\mathcal{L}_\Sigma$ with $s \in \operatorname{pre}\mathcal{V}_s$ and $\nu \in \Sigma_{uc}$. By controllability of $\operatorname{pre}\mathcal{V}'_s$ w.r.t. $\operatorname{pre}\mathcal{L}'_\Sigma$ and $\mathcal{L}_\Sigma \subseteq \mathcal{L}'_\Sigma$, we deduce that $s\nu \in \operatorname{pre}\mathcal{V}'_s$. Again by Lemma 14, we obtain $s\nu \in \operatorname{pre}(\mathcal{L}_\Sigma \cap \mathcal{V}'_s)$. Hence, $\operatorname{pre}\mathcal{V}_s$ is controllable w.r.t. $\operatorname{pre}\mathcal{L}_\Sigma$. Regarding relative closedness, observe that $(\operatorname{clo}\mathcal{V}_s) \cap \mathcal{L}_\Sigma \subseteq (\operatorname{clo}\mathcal{V}'_s) \cap \mathcal{L}'_\Sigma \cap \mathcal{L}_\Sigma = \mathcal{V}'_s \cap \mathcal{L}_\Sigma = \mathcal{V}_s$. □

As our main result, we proved, that the relevant plant properties P1–P3 of $\mathcal{L}$ are retained under closed-loop composition with $\mathcal{H}$ and, thus, also satisfied by $\mathcal{L}^H$. Moreover, the solutions of $(\Sigma^H, p^{\omega}_{ce}\mathcal{E}, \mathcal{E}^H)$ possess both properties C1 and C2 and, thus, also solve the actual problem $(\Sigma^H, \mathcal{L}^H, \mathcal{E}^H)$. In summary, Theorem 11 and Theorem 12 formally justify the hierarchical controller design as proposed by Figure 2.

5. CONCLUSION

In this paper, we discussed a closed-loop configuration with external signals, where plant and controller dynamics are represented as ω-languages. Based on Willems' notion of input-output systems, we identified the requirements P1–P3 for the plant behaviour, such that controller synthesis can be based on an abstraction while maintaining liveness and safety properties for the actual closed-loop. We have shown that the requirements P1–P3 are preserved under closed-loop composition, and, hence, that the closed-loop can again serve as a plant model. This leads to a hierarchical control architecture, in which we repeatedly design a controller, derive the closed-loop and use the specification as an abstraction for the subsequent controller design. In contrast to earlier work, e.g. (Perk et al., 2006), we treat more general liveness properties possessed by the plant or required by the specification.

Ongoing work focuses on decentralized control architectures, involving dependencies between subsystems, and the development of algorithms for the practical solution of further synthesis problems. Besides, we address the utilization of the presented configuration in the context of industrial applications. Special attention is paid to modular and reusable plant models and specifications, e.g., for production or transportation systems.

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